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TWO-DIMENSIONAL AND AXISYMMETRIC NONSTATIONARY GAS FLOWS WITH STRONG SHOCK WAVES

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We propose a way of obtaining the second and successive approximations in constructing a solution by Chernyi's method [1-3]. Limiting nonstationary flows of an inviscid gas were studied in [4, 5].

1. We consider the self-similar motion of a gas behind a strong shock wave propagating according to the law

$$x = N_0 t f_1(s), \quad y = N_0 t f_2(s) \tag{1.1}$$

Here x and y are Cartesian coordinates, N_0 is a characteristic velocity of displacement of the shock wave front, and t is time. If we let s denote arc length of the wave front in the plane of self-similar variables, then the functions f_1 and f_2 must satisfy the condition $f_1'^2 + f_2'^2 = 1$. In the axisymmetric case we take the x, y plane to be a meridional plane with the x-axis as the axis of symmetry.

We assume that all the hydrodynamic characteristics of the flow depend on two variables ξ and η. We write the gasdynamic equations in Lagrangian variables; by virtue of self-similarity, these variables are introduced in the form

$$x = N_0 t \xi(\mu, \psi), \quad y = N_0 t \eta(\mu, \psi)$$
$$\left(\xi = \frac{x}{N_0 t}, \quad \eta = \frac{y}{N_0 t}, \quad \mu = \frac{t_0}{t}\right)$$

Here t_0 and ψ define, respectively, the time instant and the polar angle of the point at which a gas particle enters the shock layer. Then, combining the two equations of motion and the equation of continuity, we obtain the system

$$\mu^2(\xi^{\cdot}\xi^{\cdot\cdot} + \eta^{\cdot}\eta^{\cdot\cdot}) = -\tau \frac{\partial p}{\partial \mu} \tag{1.2}$$
$$\mu^2\left(\xi^{\cdot\cdot}\frac{\partial \xi}{\partial \psi} + \eta^{\cdot\cdot}\frac{\partial \eta}{\partial \psi}\right) = -\tau \frac{\partial p}{\partial \psi}$$
$$\frac{\partial \eta}{\partial \psi}\frac{\partial p}{\partial \mu} - \eta^{\cdot}\frac{\partial p}{\partial \psi} = -\frac{\mu^{\nu+3}}{\eta^{\nu}}\xi^{\cdot\cdot}C(\psi)$$
$$\frac{\partial i}{\partial \mu} = \tau \frac{\partial p}{\partial \mu}, \qquad \tau = \tau(i, p)$$

Here τ is the reciprocal of the dimensionless density, p and i are the dimensionless pressure and enthalpy of the gas, and $\nu = 0$ for the two-dimensional case and $\nu = 1$ for the axisymmetric case. The function $C(\psi)$ is to be determined from conditions of dynamic compatibility at the shock wave front. The velocity vector components v_x and v_y referred to N_0, are related to the new unknown functions $\xi(\mu, \psi)$ and $\eta(\mu, \psi)$ by the equations

$$v_x = \xi - \mu\xi^{\cdot}, \quad v_y = \eta - \mu\eta^{\cdot}$$

where the dot denotes differentiation with respect to μ.

We can use the system (1.2) for constructing a certain iteration process, where for the initial approximation it is convenient to take the limiting flow of gas, which is an exact solution of the gasdynamic equations for $\tau = 0$. In each approximation the first two equations of (1.2) form a system for the determination of the law of gas particles motion. Once this law of motion has been determined, the pressure of gas is obtained from the third of the equations (1.2) and the enthalpy from the fourth equation. After determining τ from a knowledge of the pressure and enthalpy in the last of the equations (1.2), we can proceed to find the law of motion in the successive approximation from the nonlinear system consisting of the first two of the equations (1.2).

2. At the front of the shock wave (1.1), propagating at high speed through the quiescent gas, we write the dynamic compatibility conditions (for $\mu = 1$) in the form

$$\xi - \xi^{\cdot} = r(1 - \tau)\cos\gamma\,\cos(\psi - \gamma) \tag{2.1}$$
$$\eta - \eta^{\cdot} = r(1 - \tau)\cos\gamma\,\sin(\psi - \gamma)$$
$$p_+ = r^2(1 - \tau)\cos^2\gamma, \quad i_+ = {}^1/_2(1 - \tau^2)\, r^2\cos^2\gamma$$

Here

$$\operatorname{tg}\gamma = \frac{dr}{r\,d\psi}, \quad r^2 = f_1^2 + f_2^2, \quad \operatorname{tg}\psi = \frac{f_2(s_0)}{f_1(s_0)}$$

where γ is the angle between the radius vector and the external normal to the shock wave front (1.1) and s_0 is the value of the parameter s corresponding to the point at which the gas particle enters the shock layer.

The equations (2.1) are valid for shock waves of great intensity, i.e. to the situation in which terms of order a_0^2/N_0^2 can be neglected in comparison with the remaining terms (a_0 is the sound speed in the quiescent gas ahead of the shock wave front) [6].

Using the first two relations, we find the function $C(\psi)$:

$$C(\psi) = r^{\nu+2}(\sin\psi)^\nu \tag{2.2}$$

The form of the shock wave front $r = r(\psi)$ is subject to determination, in the process of solving the problem, from an additional condition. As an example of such a condition we can have a condition of flowing around, a condition on a piston, and so forth. In this regard a useful formula is that for the distance of a particle from the shock wave front [5] in the plane of the self-similar variables

$$\Lambda = \int_\mu^1 \frac{\tau\mu^{\nu+1}r^{\nu+2}\sin^\nu\psi}{\eta^\nu Q(\mu,\psi)}\sqrt{r^2+r'^2}\,d\mu \tag{2.3}$$

$$Q(\mu,\psi) = \frac{\partial\eta}{\partial\psi}\frac{\partial}{\partial\psi}(r\sin\psi) + \frac{\partial\xi}{\partial\psi}\frac{\partial}{\partial\psi}(r\cos\psi)$$

In some problems (for example, in the problem involving a piston) the formula (2.3) can be used to obtain the form of the shock wave front in a successive approximation without refining the law of motion of the particles.

3. We assume that the solution of the system (1.2) is representable in the form $q = q_0 + \varepsilon q_1$, where ε is a characteristic value of the quantity τ $(\tau_0 = 0)$. Then from the first two of the equations (1.2) we obtain the following system of equations for determining the law of motion of the particles in the zero approximation (the limiting flow):

$$\xi_0^{\cdot}\xi_0^{\cdot\cdot} + \eta_0^{\cdot}\eta_0^{\cdot\cdot} = 0, \qquad \xi_0^{\cdot\cdot}\frac{\partial\xi_0}{\partial\psi} + \eta_0^{\cdot\cdot}\frac{\partial\eta_0}{\partial\psi} = 0 \tag{3.1}$$

From the continuity equation we have $\eta_0 = f(\xi_0)$, where the form of the function f is determined by the form of the shock wave (1.1). Then the system of equations (3.1) reduces to a single equation

$$\xi_0^{\cdot\cdot}(1+f'^2) + f'f''(\xi_0^{\cdot})^2 = 0$$

Integrating this equation with respect to μ, we obtain

$$\int\sqrt{1+f'^2(\xi_0)}\,d\xi_0 = \mu C_1(\psi) + C_2(\psi)$$

It follows from this that the path traversed by a particle in the shock layer along the shock wave front depends linearly on μ. After determining the arbitrary functions $C_1(\psi)$ and $C_2(\psi)$ from the conditions (2.1) with $\tau = 0$, we can write the law of particle motion along the shock wave front in the form

$$\xi_0 = f_1(s), \quad \eta_0 = f_2(s), \quad s(\mu,\psi) = s_0(\psi) + (\mu - 1)\,C_1(\psi) \tag{3.2}$$

$$C_1(\psi) = \frac{r_0 r_0'}{\sqrt{r_0^2 + r_0'^2}}, \qquad s_0(\psi) = \int_0^\psi \sqrt{r_0^2 + r_0'^2}\,d\psi$$

After determining the function $r_0(\psi)$ from an additional condition, we proceed to a determination of the pressure. From the third of the equations (1.2), upon taking into account (2.2), we have

$$\frac{\partial \eta_0}{\partial \psi}\frac{\partial p_0}{\partial \mu} - \eta_0^{\cdot}\frac{\partial p_0}{\partial \psi} = -\mu^{\nu+3}\xi_0^{\cdot\cdot} r_0^{\nu+2}\left(\frac{\sin\psi}{\eta_0}\right)^{\nu} \equiv \Phi_0(\mu,\psi) \qquad (3.3)$$

We note that the pressure distributions in the shock layer in the two-dimensional and axisymmetric case differ in the limiting flow. The general solution of Eq. (3.3) can be written in the form

$$p_0(\mu,\psi) = F(s) + \int \frac{\Phi_0\, d\mu}{f_2'[s_0' + (\mu-1)C_1']}$$

As the argument of the arbitrary function we can take, instead of η_0 or s', an arbitrary quantity characterizing the position of a particle during its motion along the shock wave front. Upon determining the arbitrary function from the third of the conditions (2.1), we obtain the following expression for the pressure in the limiting case:

$$p_0(\mu,\psi) = p_{0+}(s) + \frac{1}{f_2'}\int_1^{\mu} \frac{\Phi_0\, d\mu}{s_0' + (\mu-1)C_1'} \qquad (3.4)$$

From the fourth of equations (1.2) we have

$$i_0 = i_{0+}(\varphi) = \frac{1}{2}p_{0+}(\psi) = \frac{1}{2}\frac{r_0^4}{r_0^2 + r_0'^2} \qquad (3.5)$$

We determine τ_1 from the equation of state (last relation in (1.2)) in terms of p_0 and i_0 Thus, for example, for a thermodynamically perfect gas

$$\tau_1 = \frac{2i_0(\psi)}{p_0(\mu,\psi)} = \frac{1}{p_0}\frac{r_0^4}{r_0^2 + r_0'^2} \qquad (3.6)$$

4. For determining the subsequent terms of the expansion, from the first four equations of (1.2) we obtain the system

$$\xi_0^{\cdot}\xi_1^{\cdot\cdot} + \xi_0^{\cdot\cdot}\xi_1^{\cdot} + \eta_0^{\cdot}\eta_1^{\cdot\cdot} + \eta_0^{\cdot\cdot}\eta_1^{\cdot} = -\frac{\tau_1}{\mu^2}\frac{\partial p_0}{\partial \mu} \qquad (4.1)$$

$$\frac{\partial \xi_0}{\partial \psi}\xi_1^{\cdot\cdot} + \xi_0^{\cdot\cdot}\frac{\partial \xi_1}{\partial \psi} + \frac{\partial \eta_0}{\partial \psi}\eta_1^{\cdot\cdot} + \eta_0^{\cdot\cdot}\frac{\partial \eta_1}{\partial \psi} = -\frac{\tau_1}{\mu^2}\frac{\partial p_0}{\partial \psi}$$

$$\frac{\partial \eta_0}{\partial \psi}\frac{\partial p_1}{\partial \mu} - \eta_0^{\cdot}\frac{\partial p_1}{\partial \psi} = \Phi_1(\mu,\psi), \qquad \frac{\partial i_1}{\partial \mu} = \tau_1\frac{\partial p_0}{\partial \mu}$$

Here

$$\Phi_1(\mu,\psi) = \eta_1^{\cdot}\frac{\partial p_0}{\partial \psi} - \frac{\partial \eta_1}{\partial \psi}\frac{\partial p_0}{\partial \mu} -$$
$$\mu^{\nu+3}\left(\frac{\sin\psi}{\eta_0}\right)^{\nu} r_0^{\nu+2}\left[\xi_1^{\cdot\cdot} - \nu\xi_0^{\cdot\cdot}\frac{\eta_1}{\eta_0} + (\nu+2)\xi_0^{\cdot\cdot}\frac{r_1}{r_0}\right]$$

To solve the inverse problem (i.e. where the wave front is specified) we must set

$$r_1(\psi) = r_2(\psi) = \ldots = 0$$

Gathering the terms in ε, we find the boundary conditions for the system (4.1) from the conditions (2.1)

$$\xi_1^{\cdot} = A_1(\psi)r_1 + B_1(\psi)r_1' + \frac{r_0}{r_0'}\eta_0^{\cdot} \qquad (4.2)$$

$$\eta_1^{\cdot} = D_1(\psi)r_1 + E_1(\psi)r_1' - \frac{r_0}{r_0'}\xi_0^{\cdot}$$

$$p_{1+} = \frac{2r_0^3}{(r_0^2 + r_0'^2)^2}[(r_0^2 + 2r_0'^2)r_1 - r_0 r_0' r_1'] - p_{0+}$$

$$i_{1+} = \frac{r_0^3}{(r_0^2 + r_0'^2)^2}[(r_0^2 + 2r_0'^2)r_1 - r_0 r_0' r_1']$$

Here A_1, B_1, D_1 and E_1 are known functions of ψ. Combining the first two of the equations (4.1), and then integrating the first of them, taking the first two of the relations (4.2) into account, we obtain a system of two first order equations

$$\xi_0^{\cdot}\xi_0^{\cdot\cdot}\frac{\partial\xi_1}{\partial\psi}+\xi_0^{\cdot}\eta_0^{\cdot}\frac{\partial\eta_1}{\partial\psi}-\xi_0^{\cdot\cdot}\frac{\partial\xi_0}{\partial\psi}\xi_1^{\cdot}-\eta_0^{\cdot\cdot}\frac{\partial\xi_0}{\partial\psi}\eta_1^{\cdot}=\frac{\tau_1}{\mu^2}\left(\frac{\partial\xi_0}{\partial\psi}\frac{\partial p_0}{\partial\mu}-\xi_0^{\cdot}\frac{\partial p_0}{\partial\psi}\right) \qquad (4.3)$$

$$\xi_0^{\cdot}\xi_1^{\cdot}+\eta_0^{\cdot}\eta_1^{\cdot}=\int_\mu^1\frac{\tau_1}{\mu^2}\frac{\partial p_0}{\partial\mu}d\mu+V_1(\psi)$$

for the determining the law of particle motion in a successive approximation. We can reduce the system (4.3) to a single second order equation; for example, for the function $\eta_1(\mu,\psi)$ we have

$$\frac{\partial^2\eta_1}{\partial\mu\,\partial\psi}-\frac{1}{\xi_0^{\cdot}}\frac{\partial\xi_0}{\partial\psi}\frac{\partial^2\eta_1}{\partial\mu^2}-\frac{\eta_0^{\cdot\cdot}}{\eta_0^{\cdot}}\frac{\partial\eta_1}{\partial\psi}+\left[\eta_0^{\cdot\cdot}\frac{(\xi_0^{\cdot})^2-(\eta_0^{\cdot})^2}{(\xi_0^{\cdot})^3\eta_0^{\cdot}}\frac{\partial\xi_0}{\partial\psi}-\frac{1}{\xi_0^{\cdot}}\frac{\partial^2\xi_0}{\partial\mu\,\partial\psi}\right]\frac{\partial\eta_1}{\partial\mu}=\frac{\xi_0^{\cdot}\eta_0^{\cdot}}{(\xi_0^{\cdot})^2+(\eta_0^{\cdot})^2}\left[\frac{\partial\varphi_1}{\partial\psi}-\frac{\partial}{\partial\mu}\left(F_1-\frac{\varphi_1}{\xi_0^{\cdot}}\frac{\partial\xi_0}{\partial\psi}\right)\right] \qquad (4.4)$$

$$F_1(\mu,\psi)=\frac{\tau_1}{\mu^2\xi_0^{\cdot}\xi_0^{\cdot\cdot}}\left(\frac{\partial\xi_0}{\partial\psi}\frac{\partial p_0}{\partial\mu}-\xi_0^{\cdot}\frac{\partial p_0}{\partial\psi}\right)$$

$$\varphi_1(\mu,\psi)=\frac{1}{\xi_0^{\cdot}}\left[\int_\mu^1\frac{\tau_1}{\mu^2}\frac{\partial p_0}{\partial\mu}d\mu+V_1(\psi)\right],\qquad V_1(\psi)=\frac{r_0'^4r_0r_1}{(r_0^2+r_0'^2)^2}$$

By changing from the independent variables μ and ψ to the variables ξ_0 and ψ, we reduce Eq. (4.4) to the canonical form

$$\frac{\partial^2\eta_1}{\partial\xi_0\partial\psi}-\frac{\eta_0^{\cdot\cdot}}{\xi_0^{\cdot}\eta_0^{\cdot}}\frac{\partial\eta_1}{\partial\psi}=L_1(\xi_0,\psi) \qquad (4.5)$$

$$L_1(\xi_0,\psi)=\frac{\eta_0^{\cdot}}{(\xi_0^{\cdot})^2+(\eta_0^{\cdot})^2}\left[\frac{\partial\varphi_1}{\partial\psi}-\frac{\partial}{\partial\mu}\left(F_1-\frac{\varphi_1}{\xi_0^{\cdot}}\frac{\partial\xi_0}{\partial\psi}\right)\right],\qquad \xi_0=\xi_0(\mu,\psi)$$

The general solution of (4.5) can be written in the form

$$\eta_1(\xi_0,\psi)=\int\eta_0^{\cdot}C_3(\psi)\,d\psi+\int\eta_0^{\cdot}K_1(\xi_0,\psi)\,d\psi+C_4(\xi_0) \qquad (4.6)$$

$$K_1(\xi_0,\psi)=\int\frac{L_1(\xi_0,\psi)}{\eta_0^{\cdot}}d\xi_0$$

To determine the function $\xi_1(\xi_0,\psi)$ we change over to the variables ξ_0 and ψ in the first equation of the system (4.3)

$$\frac{\partial\xi_1}{\partial\psi}-\frac{\xi_0^{\cdot}}{\eta_0^{\cdot}}\frac{\partial\eta_1}{\partial\psi}=F_1(\xi_0,\psi)$$

Integrating with respect to ψ, we obtain

$$\xi_1(\xi_0,\psi)=\int F_1(\xi_0,\psi)\,d\psi+\int\xi_0^{\cdot}[C_3(\psi)+K_1(\xi_0,\psi)]\,d\psi+C_5(\xi_0) \qquad (4.7)$$

For the determination of the three arbitrary functions $C_3(\psi)$, $C_4(\xi_0)$ and $C_5(\xi_0)$ we have the second equation of the system (4.3) and the two obvious conditions

$$\xi_1 = r_1 \cos\psi,\ \eta_1 = r_1 \sin\psi \quad \text{for} \quad \mu = 1 \tag{4.8}$$

Satisfying the conditions (4.8), we obtain

$$\xi_1 = \int_\varphi^\psi F_1(\xi_0, \psi)\, d\psi + \int_\varphi^\psi \xi_0^{\cdot} [C_3(\psi) + K_1(\xi_0, \psi)]\, d\psi + r_1 \cos\psi \tag{4.9}$$

$$\eta_1 = \int_\varphi^\psi \eta_0^{\cdot} [C_3(\psi) + K_1(\xi_0, \psi)]\, d\psi + r_1 \sin\psi,$$

$$\xi_0 = \xi_0(\varphi), \qquad \varphi = \operatorname{arc\,tg} \frac{f_2(s)}{f_1(s)}$$

from the relations (4.6) and (4.7).

We determine the function $C_3(\psi)$ from the second equation of the system (4.3). The solution of the third equation of (4.1) can be represented in the form (3.4). In the case of a thermodynamically perfect gas we have

$$i_1(\mu, \psi) = i_{1+}(\psi) + 2i_0(\psi) \ln \frac{p_0(\mu, \psi)}{p_{0+}} \tag{4.10}$$

from the last equation of the system (4.1).

In constructing each successive approximation the system of equations for the coefficients in the expansion of the unknown functions differ from the system (4.1) only in the terms of the right-hand sides and, consequently, can be integrated by the method presented here.

5. As an example we consider the problem involving the expansion of a piston, the equation for which, in the plane of the self-similar variables, we specify in the form

$$\frac{\xi^2}{(1+\alpha)^2} + \frac{\eta^2}{(1-\alpha)^2} = 1, \quad \alpha \ll 1 \tag{5.1}$$

in order to simplify the calculations. Neglecting terms of order α^2 and higher, from (5.1) we have

$$s = \varphi + \frac{\alpha}{2} \sin 2\varphi + O(\alpha^2), \quad \varphi = \operatorname{arctg} \frac{\eta}{\xi} \tag{5.2}$$

where φ is the instantaneous polar angle in the ξ, η plane.

The shock wave front will coincide in the limiting flow with the surface of the piston

$$r_0(\psi) = 1 + \alpha \cos 2\psi + O(\alpha^2) \tag{5.3}$$

Substituting (5.2) and (5.3) into the relations (3.2), we obtain the law of particle motion in the form

$$\varphi = \psi + 2\alpha(1-\mu)\sin 2\psi + O(\alpha^2) \tag{5.4}$$

$$\xi_0 = (1 + \alpha \cos 2\varphi)\cos\varphi, \quad \eta_0 = (1 + \alpha\cos 2\varphi)\sin\varphi$$

A problem concerning the motion of a nearly spherical piston was investigated in [7]; all terms depending on ε and terms of the first order of smallness in α were taken into account.

The integral in Eq. (3.4) is of order α^2. If in p_{0+} we retain terms of the second order of smallness in α we obtain

$$p_0 = 1 + 2\alpha\cos 2\varphi + \alpha^2\left[1 - 8\sin^2 2\varphi + \frac{4}{\nu+4}(\mu^{\nu+4} - 1)\sin^2 2\varphi\right] + O(\alpha^3)$$

as the final expression for the pressure in the limiting flow. From the relations (3.5) and (3.6) we obtain the following expressions for a perfect gas, for the enthalpy of the gas in the limiting flow and for the quantity τ_1 :

$$i_0 = {}^1/_2 [1 + 2\alpha \cos 2\varphi + \alpha^2 (1 - 8 \sin^2 2\varphi)] + O(\alpha^3) \qquad (5.5)$$

$$\tau_1 = 1 + \frac{4\alpha^2}{\nu+4} (1 - \mu^{\nu+4}) \sin^2 2\varphi + O(\alpha^3)$$

To obtain the successive approximation for the form of the shock wave front we use the relation (2.3) with $\mu = 0$. We obtain

$$r(\psi) = 1 + \alpha \cos 2\psi + \frac{\varepsilon}{\nu+2}\left[1 - \alpha \frac{2\nu + (5\nu+1)\cos 2\psi}{\nu+3}\right]$$

Retaining the principal terms in Eqs(4.9) and taking the relations (5.4) into account, we find the law of particle motion in the successive approximation in the following form:

$$\xi = \cos\psi \left\{1 + \varepsilon \frac{\mu^{\nu+2}}{\nu+2} + \alpha[1 - 2(3 - 2\mu)\sin^2\psi]\right\} \qquad (5.6)$$

$$\eta = \sin\psi \left\{1 + \varepsilon \frac{\mu^{\nu+2}}{\nu+2} - \alpha[1 - 2(3 - 2\mu)\cos^2\psi]\right\}$$

It is obvious that when $\mu = 0$ the particle coordinates given by the expressions (5.6) satisfy Eq. (5.1). Taking Eqs. (5.5) and (4.10) into account, we obtain

$$i = \frac{1}{2}(1 + 2\alpha\cos 2\psi) + \frac{\alpha^2}{2}[1 - 8(2 - \mu)\sin^2 2\psi] + \frac{\varepsilon}{\nu+2}\left[1 - 2\alpha\frac{\nu - (2-\nu)\cos 2\psi}{\nu+3}\right]$$

as the final expression for the enthalpy of the gas.

For the approximation considered we find the gas pressure to be

$$p(\mu, \psi) = 1 + 2\alpha\cos 2\psi + \alpha^2[1 + 8(\mu - 2)\sin^2 2\psi + \frac{4}{\nu+4}(\mu^{\nu+4} - 1)\sin^2 2\psi] + \frac{\varepsilon}{2(\nu+2)}[1 - \nu - (\nu+1)\mu^{2(\nu+2)}]$$

The pressure and enthalpy distribution along the surface of the axisymmetric piston are shown in Fig. 1 for $\alpha = 0.1$.

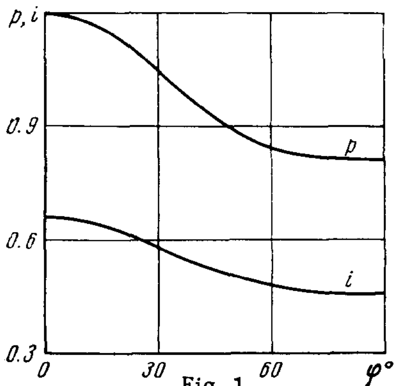

Fig. 1

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